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Quantum equivalence between the self-dual and the Maxwell–Chern–Simons models nonlinearly coupled to $U(1)$ scalar fields

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Abstract

The use of master actions to prove duality at quantum level becomes cumbersome if one of the dual fields interacts nonlinearly with other fields. This is the case of the theory considered here consisting of $U(1)$ scalar fields coupled to a self-dual field through a linear and a quadratic term in the self-dual field. Integrating perturbatively over the scalar fields and deriving effective actions for the self-dual and the gauge field we are able to consistently neglect awkward extra terms generated via master action and establish quantum duality up to cubic terms in the coupling constant. The duality holds for the partition function and some correlation functions. The absence of ghosts imposes restrictions on the coupling with the scalar fields.

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1. Introduction

The use of dual descriptions of the same physical theory is an important tool in physics as in the AdS/CFT correspondence [1]. Deep non-perturbative effects like confinement can be revealed [2] by means of duality. The usual weak coupling expansion of one theory can describe the strong coupling regime of the dual theory and vice versa as in the case of the massive Thirring and the Sine-Gordon models in (1+1) dimensions [3, 4]. In this specific case a theory with at most quartic interaction is related to a highly nonlinear theory with all powers of interacting terms. This is in fact similar to the case discussed in the present work which has its roots in the duality between the second-order Maxwell–Chern–Simons (MCS) gauge theory and the first-order self-dual (SD) model [5]. Although the equivalence between these two free theories, proved in [6] through a master action approach, is interesting in itself the most powerful applications of duality are found in interacting theories. It is therefore natural to extend the MCS/SD duality to include matter interactions [7–9], non-Abelian gauge symmetries [6, 10–13], as well as, non-commutativity [14]. In particular, we are interested here in the coupling of the self-dual field with $U(1)$ charged matter and its dual gauge theory. The authors of [7] have shown that the gauge theory dual to $U(1)$ fermions minimally coupled to the self-dual field must contain a Thirring current–current term and the minimal coupling has to be replaced by a Pauli-like coupling in the dual gauge theory. The proof, based on a master action, holds for the equations of motion and the partition function. In [8] the case of both charged fermions and scalar fields minimally coupled to the self-dual field were considered only at the classical level. In the case of scalar fields, which is considered here, we have an extra complication. Namely, the dual gauge theory contains besides a Thirring term a highly nonlinear interaction between the gauge and the matter fields through the coefficient of the Maxwell term which contains scalar fields in its denominator. The source of complicated nonlinear terms is the dependence of the $U(1)$ current on the self-dual field which is absent for fermions. In [9] we have argued that due to the lack of gauge symmetry in the self-dual model there is no need for a minimal coupling with the matter fields. Thus, we can suppress the field-dependent part of the $U(1)$ current and work with a linear coupling in the self-dual field similarly to the case of fermions where the minimal and linear couplings are the same. In this case we have been able [9] to derive the dual equivalent gauge theory through a master action and prove the dual equivalence at quantum level. That corresponds in our notation to the case $a = 0$, see (1) and (3), where the highly nonlinear terms present in the dual gauge theory (6) disappear. The aim of this work is to return to the general case $a \neq 0$ and prove the dual equivalence between $U(1)$ scalar fields nonlinearly coupled to the self-dual field and its dual gauge theory at quantum level. By calculating the functional determinant from the integral over the scalar fields until quadratic terms in the coupling we will prove the dual equivalence at, to that order, of the partition functions and some correlators thus going beyond the proof of classical equivalence for arbitrary values of a given in [8, 9] and quantum equivalence for $a = 0$ presented in [9]. Our result is a nontrivial check of the field dependence of the coefficient of the Maxwell term appearing in the dual gauge theory.

In section 2.1, starting from a master action we recall the classical equivalence of the self-dual model nonlinearly coupled to $U(1)$ scalar fields with its dual gauge theory. In section 2.2 by integrating over the matter fields perturbatively, we prove the dual equivalence of the corresponding partition functions disregarding cubic and higher terms in the coupling constant. In section 2.3 we include sources and extend the proof to correlation functions. In section 3 we analyse the spectrum of the effective action for the self-dual field regarding the presence of ghosts. In the final section we present the conclusions.

2. Dual equivalence

2.1. Equations of motion

Our starting point is the master Lagrangian suggested in [8, 9]:

$$\mathcal{L}_{\text{Master}} = \frac{\mu^2}{2} f^\mu f_\mu - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - e f^\nu J_\nu^{(0)} + \mathcal{L}_{\text{Matter}} + \frac{m}{2}\epsilon_{\alpha\beta\gamma}(f^\alpha - A^\alpha)\partial^\beta(f^\gamma - A^\gamma), \tag{1}$$

where

$$J_\nu^{(0)} = i(\phi^*\partial_\nu\phi - \phi\partial_\nu\phi^*) \tag{2}$$

$$\mu^2 = m^2 + 2ae^2\phi^*\phi \tag{3}$$

$$\mathcal{L}_{\text{Matter}} = -\phi^*\left(\Box + m_\phi^2\right)\phi. \tag{4}$$

We assume $g_{\mu\nu} = (+, -, -)$. The quantity a is a constant and the case of minimal coupling corresponds to $a = -1$. Since the gauge invariance of the master Lagrangian is guaranteed for any value of a we do not need to stick to the minimal coupling. We have shown in [9] that from the equations of motion $\delta\mathcal{L}_{\mathrm{Master}} = 0$ we can derive two sets of equations of motion $\delta\mathcal{L}_{\mathrm{M+SD}} = 0$ and $\delta\mathcal{L}_{\mathrm{M+MCS}} = 0$ where

$$\mathcal{L}_{\mathrm{M+SD}} = \frac{\mu^2}{2} f^\mu f_\mu - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - e f^\nu J_\nu^{(0)} + \mathcal{L}_{\mathrm{Matter}} \tag{5}$$

$$\mathcal{L}_{\mathrm{M+MCS}} = -\frac{m^2}{4\mu^2} F_{\alpha\beta}(A) F^{\alpha\beta}(A) + \frac{m}{2}\epsilon_{\alpha\beta\gamma} A^\alpha \partial^\beta A^\gamma - \frac{me}{\mu^2} J_\nu^{(0)} \epsilon^{\nu\alpha\beta} \partial_\alpha A_\beta - \frac{e^2}{2\mu^2} J_\nu^{(0)} J^{\nu(0)} + \mathcal{L}_{\mathrm{matter}}. \tag{6}$$

Furthermore, the equations of motion of $\mathcal{L}_{\mathrm{M+SD}}$ and $\mathcal{L}_{\mathrm{M+MCS}}$ are equivalent to each other through the dual map $f_\nu \leftrightarrow \tilde{A}_\nu$ where

$$\tilde{A}_\mu \equiv -\frac{m}{\mu^2}\epsilon_{\mu\nu\alpha}\partial^\alpha A^\nu + \frac{e}{\mu^2} J_\mu^{(0)}. \tag{7}$$

The classical equivalence holds for arbitrary values of a. Concerning the role of the minimal coupling ($a = -1$) a comment is in order. Namely, the equations of motion of $\mathcal{L}_{\mathrm{M+SD}}$ lead to $\partial_\nu\{[m^2 + 2(a+1)e^2\phi^*\phi] f^\nu\} = 0$ which works like a gauge condition assuring that the gauge field A_μ and the self-dual field f_μ have the same number of degrees of freedom for arbitrary values of a. Though it is not mandatory to fix $a = -1$, in that case we deduce the simple equation $\partial_\nu f^\nu = 0$ which appears in the free self-dual model.

2.2. *Effective actions*

In order to check duality at quantum level we start with the partition function:

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}f^\nu\mathcal{D}A^\nu \exp\left(\imath \int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{master}}\right). \tag{8}$$

The gauge field A_ν, after a translation $A_\nu \to A_\nu + f_\nu$, can be easily integrated leading to

$$\mathcal{Z} = C \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}f^\nu \exp\left(\imath \int \mathrm{d}^3x\, \mathcal{L}_{\mathrm{M+SD}}\right), \tag{9}$$

where C is a constant. On the other hand, starting from (8) and performing the translation $f_\nu \to f_\nu + \left(eJ_\nu^{(0)} - m\epsilon_{\nu\alpha\beta}\partial^\beta A^\alpha\right)/\mu^2$ we arrive at the dual theory $\mathcal{L}_{\mathrm{M+MCS}}$ plus an extra term,

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}f^\nu\mathcal{D}A^\nu \exp\left(\imath \int \mathrm{d}^3x [\mathcal{L}_{\mathrm{M+MCS}} + \mathcal{L}_{\mathrm{extra}}]\right), \tag{10}$$

where

$$\mathcal{L}_{\mathrm{extra}} = (m^2 + 2ae^2\phi^*\phi)\frac{f^\nu f_\nu}{2}. \tag{11}$$

At classical level, $\mathcal{L}_{\mathrm{extra}}$ can be dropped since its equations of motion imply $f_\nu = 0$. At quantum level, the functional integral over f_ν will be matter field dependent for $a \neq 0$ and there seems to be no simple way to disregard those potentially divergent contributions. In order to avoid such problems we have assumed in [9] the linear coupling condition $a = 0$ which allowed us to rigorously prove the dual equivalence between $\mathcal{L}_{\mathrm{M+MCS}}$ and $\mathcal{L}_{\mathrm{M+SD}}$ at quantum level including matter and vector field correlation functions. If $a = 0$ we have $\mu^2 = m^2$ and the complicated nonlinearities appearing in (6) disappear. For $a \neq 0$ we need perturbative methods. Integrating over the scalar fields in (8) we have[1]

$$\mathcal{Z} = \int \mathcal{D}f^\nu \mathcal{D}A^\nu \exp\left(\imath \int \mathrm{d}^3x\, \mathcal{L}(A, f) - \mathrm{Tr}\ln\left[-\Box - m_\phi^2 - \mathrm{i}e(\partial_\nu f^\nu + 2f^\nu\partial_\nu) + ae^2 f^\alpha f_\alpha\right]\right)$$
$$= \int \mathcal{D}f^\nu \mathcal{D}A^\nu \exp\left(\left[\imath \int \mathrm{d}^3x\, \mathcal{L}(A, f) + \frac{1}{2}\int \mathrm{d}^3k\, f^\alpha(-k) T_{\alpha\beta} f^\beta(k) + \mathcal{O}(e^3)\right]\right) \tag{12}$$

where

$$\mathcal{L}(A, f) = \frac{\mu^2}{2} f^\mu f_\mu - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma + \frac{m}{2}\epsilon_{\alpha\beta\gamma}(f^\alpha - A^\alpha)\partial^\beta(f^\gamma - A^\gamma). \tag{13}$$

The second term in the exponential is written in momentum space in terms of the Fourier transforms $f_\nu(k)$. At quadratic order in coupling we have only two Feynman integrals. A careful derivation leads to

$$T_{\alpha\beta} = 2ae^2 g_{\alpha\beta} I^{(1)} + e^2 I^{(2)}_{\alpha\beta}. \tag{14}$$

Using dimensional regularization we have obtained for the Feynman integrals:

$$I^{(1)} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{p^2 - m_\phi^2} = \mathrm{i}\frac{m_\phi}{4\pi} \tag{15}$$

$$I^{(2)}_{\alpha\beta} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{(2p+k)_\alpha (2p+k)_\beta}{\left(p^2 - m_\phi^2\right)\left[(p-k)^2 - m_\phi^2\right]} = \frac{\mathrm{i}m_\phi}{8\pi}\left[4g_{\alpha\beta} - \theta_{\alpha\beta}\left(2 + \frac{z-1}{\sqrt{z}}\ln\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] \tag{16}$$

with $z = k^2/4m_\phi^2$ and $\theta_{\alpha\beta} = g_{\alpha\beta} - k_\alpha k_\beta/k^2$. Our results for $I^{(1)}, I^{(2)}$ are in agreement with [15]. The expressions for the integrals were given in the region $0 \leqslant z \leqslant 1$. For $z < 0$ we can analytically continue the expressions. Above the pair creation threshold ($z > 1$) the integral $I^{(2)}_{\alpha\beta}$ develops a real part which will be neglected here. This is a good approximation for large m_ϕ. Note that for $a = -1$ (minimal coupling) $T_{\alpha\beta}$ becomes the vacuum polarization tensor $\Pi_{\alpha\beta}$ of scalar QED in (2+1) dimensions which is transverse $k^\alpha \Pi_{\alpha\beta} = 0 = \Pi_{\alpha\beta}k^\beta$. Back in (12) we can write down the effective master action at quadratic order:

$$\mathcal{L}_{\mathrm{Master}} = \frac{m^2 + c_2}{2} f^\alpha f_\alpha - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - \frac{c_1}{4} F_{\alpha\beta}(f) B(\Box) F^{\alpha\beta}(f) \tag{17}$$
$$+ \frac{m}{2}\epsilon_{\alpha\beta\gamma}(A - f)^\alpha \partial^\beta (A - f)^\gamma + \mathcal{O}(e^3), \tag{18}$$

where we have defined

$$c_1 = \frac{e^2}{16\pi m_\phi} \tag{19}$$

$$c_2 = \frac{(a+1)}{2\pi} e^2 m_\phi \tag{20}$$

$$B(\Box) = \frac{1}{z}\left(1 + \frac{z-1}{2\sqrt{z}}\ln\frac{1+\sqrt{z}}{1-\sqrt{z}}\right). \tag{21}$$

[1] Throughout this work a small coupling expansion is understood as an expansion in the dimensionless constant e^2/m_ϕ.

Since the quadratic terms in the self-dual field in (18) are scalar field independent, the integration over f_ν does not generate unwanted extra terms as before and we derive, after expanding in the coupling, the non-local MCS theory:

$$\mathcal{L}^{(e^2)}_{\mathrm{NL-MCS}} = -\frac{m}{2}A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu + \frac{1}{4}F_{\mu\nu}(A)[-1 + c_2 + c_1\,\square\, B(\square)]F^{\mu\nu}(A). \tag{22}$$

On the other hand, if we believe that (6) is the correct dual gauge theory obtained from the integration over f_ν in (8), neglecting the extra term (11), then it should be possible to derive (22) directly from (6) by integrating over the scalar fields to the quadratic order in the coupling. If we restrict $\mathcal{L}_{\mathrm{M+MCS}}$ to the same order e^2 and introduce an auxiliary vector field B_ν to lower the nonlinearity of the Thirring term, the partition function associated with (6) will be given by

$$\mathcal{Z}_{\mathrm{M+MCS}} = \int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}B^\nu\mathcal{D}A^\nu \exp\left(\imath\int \mathrm{d}^3x\,\mathcal{L}^{(e^2)}_{\mathrm{M+MCS}}\right), \tag{23}$$

where

$$\mathcal{L}^{(e^2)}_{\mathrm{M+MCS}} = -\frac{m}{2}A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu + F_{\mu\nu}(A)F^{\mu\nu}(A)\left(-\frac{1}{4} + \frac{ae^2}{2m^2}\phi^*\phi\right) + \frac{B^\nu B_\nu}{2} - \frac{e}{m}J^{(0)}_\nu(B^\nu + \epsilon^{\nu\alpha\beta}\partial_\alpha A_\beta) + \mathcal{L}_{\mathrm{matter}}. \tag{24}$$

We have expanded the coefficient of the Maxwell term up to the second order in coupling. Integrating over the scalar fields, using (15) and (16) and Gaussian integrating over B_ν we obtain

$$\mathcal{Z}_{\mathrm{M+MCS}} = D\int \mathcal{D}A^\nu \exp\left(\imath\int \mathrm{d}^3x\,\mathcal{L}_{\mathrm{eff}}\right) \tag{25}$$

with D being a constant. The effective Lagrangian turns out to match (22) after expansion up to the quadratic order in the coupling:

$$\mathcal{L}^{(e^2)}_{\mathrm{eff}} = -\frac{m}{2}A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu + F_{\mu\nu}(A)\left[\frac{ae^2m_\phi}{8\pi m^2} - \frac{1/4}{1 + \frac{e^2m_\phi}{2\pi m^2} + \frac{c_1\,\square\, B(\square)}{m^2}}\right]F^{\mu\nu}(A) + \mathcal{O}(e^3) = \mathcal{L}^{(e^2)}_{\mathrm{NL-MCS}} + \mathcal{O}(e^3). \tag{26}$$

Therefore, using (8), (9) and (23), (25), (26) we have shown that the partition functions corresponding to the classically equivalent theories $\mathcal{L}_{\mathrm{M+MCS}}$ and $\mathcal{L}_{\mathrm{M+SD}}$ are equivalent to the order e^2 up to an overall constant. In other words, the extra term (11) can be completely disregarded to the above order, although $a \neq 0$.

Now we have an interesting remark about the case of N_f flavours of scalar fields. This case requires $e \to e/\sqrt{N_f}$ in our starting Lagrangian (1) which would imply $\mu^2 \to m^2 + (2ae^2/N_f)\sum_{j=1}^{N_f}\phi_j\phi_j^*$. It is easy to convince oneself that the integration over the N_f scalar fields could be done exactly in the limit $N_f \to \infty$ resulting precisely in our quadratic master action (18). After integration over the self-dual field we would obtain

$$\mathcal{L}_{\mathrm{NL-MCS}}(N_f \to \infty) = -\frac{m}{2}A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu - \frac{m^2}{4}F_{\mu\nu}(A)\frac{1}{[m^2 + c_2 + c_1\,\square\, B(\square)]}F^{\mu\nu}(A). \tag{27}$$

On the other hand, we should be able to derive the Lagrangian above starting from the dual gauge theory $\mathcal{L}_{\mathrm{M+MCS}} + \mathcal{L}_{\mathrm{extra}}$, which now contains $1/\mu^2 = 1\big/\big[m^2 + (2ae^2/N_f)\sum_{j=1}^{N_f}\phi_j\phi_j^*\big]$ in front of the Maxwell term, by taking $N_f \to \infty$. It turns out that this is not trivial since the term $\sum_{j=1}^{N_f}\phi_j\phi_j^*/N_f$ which appears in $1/\mu^2$ is *a priori* not small at $N_f \to \infty$. Thus, the duality allows us to carry out a sum in (6) of infinite terms of the same order in $1/N_f$ which allows an exact solution of $\mathcal{L}_{\mathrm{M+MCS}} + \mathcal{L}_{\mathrm{extra}}$ in the limit $N_f \to \infty$.

2.3. Correlation functions

Returning to the case $N_f = 1$, by introducing sources and comparing correlation functions we will show that the dual map $f_\nu \leftrightarrow \tilde{A}_\nu$ holds at quantum level. As in [16] we add sources for the dual field $\tilde{A}_\nu$, given in (7). Defining $\mathcal{D}M \equiv \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}f^\nu\mathcal{D}A^\nu$, we deduce

$$\mathcal{Z}(J) = \int \mathcal{D}M \exp\left(\imath \int \mathrm{d}^3x[\mathcal{L}_{\mathrm{master}} + J^\nu\tilde{A}_\nu]\right) = \int \mathcal{D}M \times \exp\left(\imath\int \mathrm{d}^3x\left[\mathcal{L}_{\mathrm{master}} + f_\nu J^\nu + \frac{J_\nu J^\nu}{2\mu^2}\right]\right) \tag{28}$$

$$= C\int \mathcal{D}\phi\mathcal{D}\phi^*\mathcal{D}f^\nu \exp\left(\imath\int \mathrm{d}^3x\left[\mathcal{L}_{\mathrm{M+SD}} + f_\nu J^\nu + \frac{J_\nu J^\nu}{2\mu^2}\right]\right). \tag{29}$$

In (28) we have simply made a translation $f_\nu \to f_\nu + J_\nu/\mu^2$ while to get (29) we did $A_\nu \to A_\nu + f_\nu$ and integrated over the gauge field producing the overall constant C. Deriving $\ln\mathcal{Z}(J)$ with respect to the sources we can prove the following identity for connected correlation functions:

$$\left\langle \tilde{A}_{\nu_1}(x_1)\cdots\tilde{A}_{\nu_n}(x_n)\right\rangle_{\mathrm{Master}} = \left\langle f_{\nu_1}(x_1)\cdots f_{\nu_n}(x_n)\right\rangle_{\mathrm{SD+M}} + \mathrm{CT}, \tag{30}$$

where CT stands for contact terms. For instance, for the two point functions we have

$$\left\langle \tilde{A}_{\nu_1}(x_1)\tilde{A}_{\nu_2}(x_2)\right\rangle_{\mathrm{Master}} = \left\langle f_{\nu_1}(x_1)f_{\nu_2}(x_2)\right\rangle_{\mathrm{SD+M}} + g_{\nu_1\nu_2}\delta(x_1 - x_2)\left\langle\frac{1}{\mu^2}\right\rangle_{\mathrm{SD+M}}. \tag{31}$$

From (30) we see that whatever is the gauge theory obtained from the master action by integration over f_ν, the correlation functions of $\tilde{A}_\nu$ in such theory will coincide with the correlation functions of f_ν in $\mathcal{L}_{\mathrm{M+SD}}$ for arbitrary values of a up to contact terms. Due to the difficulties related with the integration over f_ν, see (10), we have to stick once again to perturbative calculations in order to relate the left-hand side of (30) with the theory (6). By repeating the steps which have led us from (8) to (22) now in the presence of sources we have

$$\int \mathcal{D}f^\nu\mathcal{D}\phi\mathcal{D}\phi^* \exp\left(\imath\int \mathrm{d}^3x[\mathcal{L}_{\mathrm{master}} + J^\nu\tilde{A}_\nu]\right) = \exp\left(\imath\int \mathrm{d}^3x\,\mathcal{L}^{(e^2)}(J)\right), \tag{32}$$

where

$$\mathcal{L}^{(e^2)}(J) = \mathcal{L}^{(e^2)}_{\mathrm{NL-MCS}} + J^\mu\left[\frac{e^2m_\phi}{4\pi m^4}g_{\mu\nu} + \frac{c_1\,\Box\,B(\Box)}{2m^4}\theta_{\mu\nu}\right]J^\nu + J^\mu\left[\frac{1}{m} - \frac{e^2m_\phi(a+1)}{2\pi m^3} + \frac{c_1\,\Box\,B(\Box)}{m^3}\right]\epsilon_{\mu\alpha\nu}\partial^\alpha A^\nu + \mathcal{O}(e^3). \tag{33}$$

In the expression (33) we have used $\theta_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\partial_\beta/\Box$. On the other hand, integrating over the matter fields disregarding terms of order $\mathcal{O}(e^3)$, as in the derivation of (26) from (23), we can deduce

$$= \int \mathcal{D}\phi\mathcal{D}\phi^* \exp\left(\imath\int \mathrm{d}^3x\left[\mathcal{L}^{(e^2)}_{\mathrm{M+MCS}} + J^\nu\tilde{A}_\nu + \mathcal{O}(e^3)\right]\right) = \exp\left(\imath\int \mathrm{d}^3x\,\mathcal{L}^{(e^2)}(J)\right). \tag{34}$$

From (32) and (34) we derive

$$\left\langle \tilde{A}_{\nu_1}(x_1)\cdots\tilde{A}_{\nu_n}(x_n)\right\rangle_{\mathrm{M+MCS}} = \left\langle \tilde{A}_{\nu_1}(x_1)\cdots\tilde{A}_{\nu_n}(x_n)\right\rangle_{\mathrm{Master}} + \mathcal{O}(e^3). \tag{35}$$

From (30) and (35) we conclude

$$\left\langle \tilde{A}_{\nu_1}(x_1)\cdots\tilde{A}_{\nu_n}(x_n)\right\rangle_{\mathrm{M+MCS}} = \left\langle f_{\nu_1}(x_1)\cdots f_{\nu_n}(x_n)\right\rangle_{\mathrm{SD+M}} + \mathrm{CT} + \mathcal{O}(e^3). \tag{36}$$

Therefore, the mapping $f_\nu \leftrightarrow \tilde{A}_\nu$ also holds at quantum level, at least if we neglect terms of order e^3.

For $a = 0$ we have shown in [9] that matter field correlators in $\mathcal{L}_{\text{M+MCS}}$ and in $\mathcal{L}_{\text{M+SD}}$ are equal since no integration over matter fields is necessary to go from $\mathcal{L}_{\text{M+MCS}}$ to $\mathcal{L}_{\text{M+SD}}$ via master action. For $a \neq 0$, had we added scalar field sources in (28), that is, instead of $J^\nu \tilde{A}_\nu$ we had $J^\nu \tilde{A}_\nu + \psi\phi + \psi^*\phi^*$, since no scalar field integration is carried out to obtain (29), we would be able to prove that scalar correlators in $\mathcal{L}_{\text{Master}}$ and in $\mathcal{L}_{\text{M+SD}}$ would be equal which is the analogous of (30) for pure scalar field correlators. However, since the matter fields are integrated over perturbatively in (32) and (34), the reader can check that such integral in the presence of the sources ψ, ψ^* would generate terms of the type $\psi\left(\Box + m_\phi^2\right)^{-1} a e^2 f^2 \psi*$ thus leading to divergences in the integral over the self-dual field which has a delta function propagator, as commented in [9]. Such divergences would invalidate our perturbative integration over the scalar fields. Therefore, the connection between the scalar field correlators in the $\mathcal{L}_{\text{Master}}$ and those correlators in $\mathcal{L}_{\text{M+MCS}}$ is more complicated and we are not able to prove equivalence with the corresponding correlators in $\mathcal{L}_{\text{M+SD}}$, not even at quadratic order in the coupling.

3. Spectrum

After a translation $A_\nu \to A_\nu + f_\nu$ in (18) we can integrate over the gauge field yielding an effective non-local self-dual model:

$$\mathcal{L}_{\text{NL-SD}} = \frac{m^2 + c_2}{2} f^\alpha f_\alpha - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - \frac{c_1}{4} F_{\alpha\beta}(f) B(\Box) F^{\alpha\beta}(f). \tag{37}$$

The effect of the matter fields determinant, up to the considered order, was to produce another mass term for the self-dual field plus a non-local Maxwell term.

Now in order to verify whether our quadratic truncation furnishes sensible theories we check the spectrum of both quadratic theories (37) and (22). It is a general result, see [17], that due to the fact that (37) and (22) are connected via a Chern–Simons mixing term, see (18), the propagators coming from both theories will have the same pole structure except for a non-physical, gauge-dependent, massless pole $k^2 = 0$ associated with the Chern–Simons term which will appear in the propagator of the gauge field as one can explicitly check from (22). Consequently, we only need to check the spectrum of (37). In the large mass limit $m_\phi \to \infty$ $(z \to 0)$ using a derivative expansion $B(\Box) = 2/3 + \mathcal{O}\left(-\Box/m_\phi^2\right)$ we recover a local theory of the Maxwell–Chern–Simons–Proca type:

$$\begin{aligned}\mathcal{L}^{(e^2)}_{\text{NL-SD}} = &\left[m^2 + \frac{(a+1)}{2\pi} e^2 m_\phi\right] \frac{f^\alpha f_\alpha}{2} - \frac{m}{2}\epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma \\ &- \frac{e^2}{96\pi m_\phi} F_{\alpha\beta}(f) F^{\alpha\beta}(f) + \mathcal{O}\left(\frac{1}{m_\phi^3}\right).\end{aligned} \tag{38}$$

It is possible to show [17, 18] that the Maxwell–Chern–Simons–Proca theory is free of ghosts whenever the coefficient of the Maxwell term is non-positive and the coefficient of the Proca term is non-negative. This requires $a \geqslant a^* \equiv -1 - (2\pi m^2)/(e^2 m_\phi)$ which includes the linear coupling $a = 0$ and the minimal coupling $a = -1$. If the condition $a \geqslant a^*$ is satisfied we have a perfectly well-defined theory with two massive physical poles. We note that for $a \neq -1$ the limit $m_\phi \to \infty$ only makes sense if we assume the scaling $e^2 \sim \alpha/m_\phi$ where α is some constant with mass square dimension, after which $\mathcal{L}^{(e^2)}_{\text{NL-SD}}$ becomes a self-dual model with a modified mass due to the matter fields determinant. At leading order the Maxwell term is neglected and we end up with just one massive pole if $a \neq a^*$. In the case $a = a^*$ we have, quite surprisingly, a gauge theory. The gauge non-invariance of the non-minimal coupling with the scalar fields cancels the mass term of the self-dual model. In this special case the duality relates two gauge theories. On one hand we have a local MCS theory, see (38) without the Proca term, on the other hand (22) becomes for $c_2 = -m^2$ and $B(\Box) = 2/3$ a non-local MCS Lagrangian: $(m/2)A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu - (6\pi m^2 m_\phi/e^2)F_{\alpha\beta}(1/\Box)F^{\alpha\beta}$. In particular, we have the coupling e^2/m_ϕ on one side and m_ϕ/e^2 on the dual side which is typical for dual theories. For the minimal coupling $a = -1$, in the limit $m_\phi \to \infty$, (37) becomes at leading order a pure self-dual model with the same original mass as before the coupling to the scalar field. In summary, both effective theories (37) and (22) are perfectly well-defined dual field theories with the same particle content as far as $a \geqslant a^*$.

4. Conclusion

The most useful applications of duality concern interacting theories. It is specially interesting to connect complicated nonlinear theories with simpler dual models. Here we have shown how a perturbative integration over part of the degrees of freedom can help us to find such connections at quantum level. Explicitly, we have demonstrated, by integrating the scalar fields to the order e^2, that the classical map $f_\mu \leftrightarrow \tilde{A}_\mu$ holds also at quantum level at least perturbatively up to terms of order $\mathcal{O}(e^3)$. The quadratic effective free theories obtained lead to sensible quantum field theories for a large range of the couplings which includes the linear ($a = 0$) and the minimal ($a = -1$) couplings. In particular, although there is some simplification at classical level for the minimal coupling, there are apparently no physical requirements to force us to assume such coupling at quantum level to the order examined here.

We remark that one of the difficulties in relating nonlinear theories through a master action is the presence of extra terms like (11) which have been consistently neglected here but can possibly play a role at higher orders in the coupling constant which demands the inclusion of higher corrections to the scalar fields determinant. A complete proof of quantum duality between SD and MCS theories nonlinearly coupled to $U(1)$ scalar fields requires perhaps a non-perturbative analysis of the matter correlators in both theories. It is tempting to blame the bad infrared behaviour of the self-dual field for the infinities related with the extra term (11). The non-Abelian and the non-commutative cases of SD/MCS duality suffer from problems alike, i.e., the quadratic terms in the self-dual field do not have constant coefficients which makes the integral over those fields complicated.

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